MATHEMATICS   **9794/03**

Paper 3 Applications of Mathematics   **May/June 2012**

2 hours

Additional Materials: Answer Booklet/Paper
                         Graph Paper
                         List of Formulae (MF20)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.

Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

Where a numerical value for the acceleration due to gravity is needed, use $10 \, \text{m s}^{-2}$.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 80.

You are advised to spend no more than 1 hour on Section A and 1 hour on Section B.

This document consists of **5** printed pages and **3** blank pages.

[Turn over

Section A: Probability (40 marks)

You are advised to spend no more than 1 hour on this section.

1   The heights in centimetres of 10 young women were measured and are given below.

   140   145   162   174   153   167   147   151   148   156

   Calculate the mean height of these women and show that the standard deviation is approximately
   10 cm.   [4]

2   A bag contains four black balls and one white ball. A man chooses a ball at random. If it is a black
   ball, he replaces it and chooses another at random. If he chooses the white ball, he stops.

   (i) Name the probability distribution which models this situation.   [1]

   (ii) Calculate the probability that he will make exactly three attempts before he stops.   [2]

   (iii) Calculate the probability that he will make fewer than three attempts before he stops. [2]

3   The lengths of snakes on a tropical island were measured and found to be normally distributed with a
   mean of 160 cm and a standard deviation of 6 cm. Find the probability that a randomly selected snake
   has a length of less than 170 cm.   [4]

4   In one department of a firm, four employees are selected for promotion from a staff of eighteen.

   (i) In how many ways can four employees be selected?   [2]

   It is known that throughout the firm 5% of those selected for promotion decline it.

   (ii) If 100 employees are randomly selected for promotion in the firm, calculate the number expected
   to decline promotion.   [1]

   (iii) If 20 employees are selected at random for promotion, use the binomial distribution to find the
   probability that fewer than five employees will decline promotion.   [3]

5   In an archery competition, competitors are allowed up to three attempts to hit the bulls-eye. No one
   who succeeds may try again.

   45% of those entering the competition hit the bulls-eye first time. For those who fail to hit it the first
   time, 60% of those attempting it for the second time succeed in hitting it. For those who fail twice,
   only 15% of those attempting it for the third time succeed in hitting it. By drawing a tree diagram, or
   otherwise,

   (i) find the probability that a randomly chosen competitor fails at all three attempts,   [2]

   (ii) find the probability that a randomly chosen competitor fails at the first attempt but succeeds at
   either the second or third attempt,   [3]

   (iii) find the probability that a randomly chosen competitor succeeds in hitting the bulls-eye, [2]

   (iv) find the probability that a randomly chosen competitor requires exactly two attempts given that
   the competitor is successful.   [3]

6 James plays an arcade game. Each time he plays, he puts a £1 coin in the slot to start the game. The possible outcomes of each game are as follows:

James loses the game with a probability of 0.7 and the machine pays out nothing,

James draws the game with a probability of 0.25 and the machine pays out a £1 coin,

James wins the game with a probability of 0.05 and the machine pays out ten £1 coins.

The outcomes can be modelled by a random variable X representing the number of £1 coins gained at the end of a game.

(i) Construct a probability distribution table for X. [2]

(ii) Show that $E(X) = -0.25$ and find $Var(X)$. [4]

James starts off with 10 £1 coins and decides to play exactly 10 games.

(iii) Find the expected number of £1 coins that James will have at the end of his 10 games. [2]

(iv) Find the probability that after his 10 games James will have at least 10 £1 coins left. [3]

Section B: Mechanics (40 marks)

You are advised to spend no more than 1 hour on this section.

7
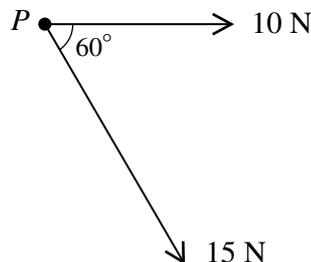
The diagram shows two forces of magnitudes 10 N and 15 N acting in a horizontal plane on a particle P.

(i) Find the component of the 15 N force which is parallel to the 10 N force. [2]

(ii) Write down the component of the 15 N force which is perpendicular to the 10 N force. [1]

(iii) Hence, or otherwise, calculate the magnitude and direction of the resultant force on P. [4]

8 A crane lifts a crate of mass 20 kg using a light inextensible cable. The crate starts from rest and ascends 10 metres in 4 seconds during which time a constant tension of T N is applied in the cable. Find the value of T. [4]

9
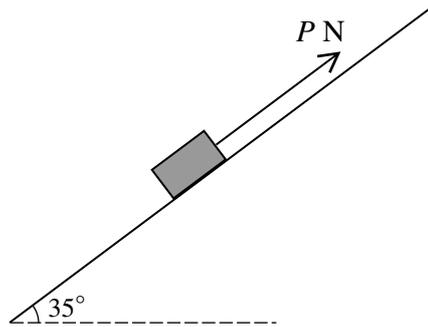
The diagram shows a block of wood, weighing $100\,\text{N}$, at rest on a rough plane inclined at $35°$ to the horizontal. The coefficient of friction between the block and the plane is 0.2. A force of $P\,\text{N}$ acts on the block up the slope.

    (i) Find the maximum possible value of the friction acting on the block. [2]

   (ii) Given that the block is on the point of moving *up* the slope, find P. [2]

  (iii) Given that the block is on the point of moving *down* the slope, find P. [2]

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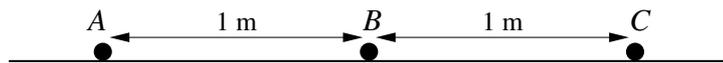
Three particles A, B and C, having masses $1\,\text{kg}$, $2\,\text{kg}$ and $5\,\text{kg}$, respectively, are placed 1 metre apart in a straight line on a smooth horizontal plane (see diagram). The particles B and C are initially at rest and A is moving towards B with speed $14\,\text{m\,s}^{-1}$. The coefficient of restitution between each pair of particles is 0.5.

    (i) Find the velocity of B immediately after the first impact and show that A comes to rest. [4]

   (ii) Show that B reverses direction after an impact with C. [3]

  (iii) Find the distance between B and C at the instant that B collides with A for the second time. [3]

11 A particle P of mass $2\,\text{kg}$ can move along a line of greatest slope on the smooth surface of a wedge which is fixed to the ground. The sloping face OA of the wedge has length 10 metres and is inclined at $30°$ to the horizontal (see Fig. 1). P is fired up the slope from the lowest point O, with an initial speed of $20\,\text{m s}^{-1}$.

Fig. 1

(i) Find the time taken for P to reach A and show that the speed of P at A is $10\sqrt{3}\,\text{m s}^{-1}$. [6]

After P has reached A it becomes a projectile (see Fig. 2).
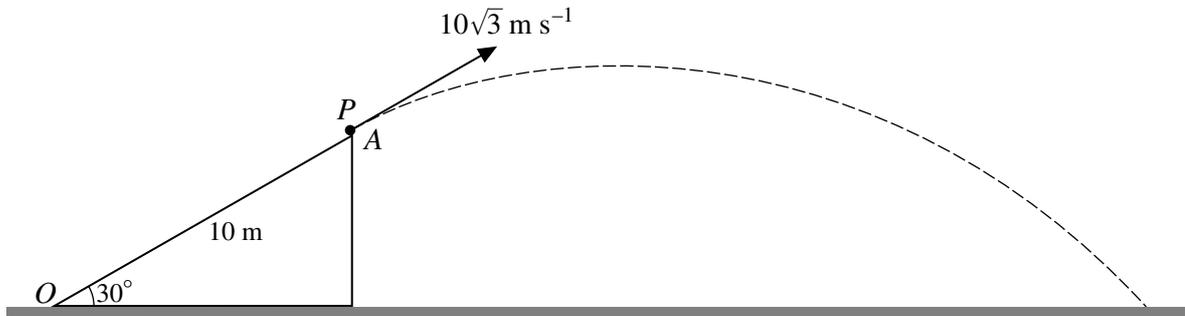
Fig. 2

(ii) Find the total horizontal distance travelled by P from O when it hits the ground. [7]

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