Numerical Solutions of Equations

Question Paper 5

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Numerical Solutions of Equations
Sub Topic	
Booklet	Question Paper 5

Time Allowed: **54 minutes**

Score: **/45**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1 The constant a, where $a > 1$, is such that $\int_1^a \left(x + \frac{1}{x}\right) dx = 6$.

(i) Find an equation satisfied by a, and show that it can be written in the form

$$a = \sqrt{(13 - 2\ln a)}.$$ [5]

(ii) Verify, by calculation, that the equation $a = \sqrt{(13 - 2\ln a)}$ has a root between 3 and 3.5. [2]

(iii) Use the iterative formula

$$a_{n+1} = \sqrt{(13 - 2\ln a_n)},$$

with $a_1 = 3.2$, to calculate the value of a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

2 The sequence of values given by the iterative formula

$$x_{n+1} = \frac{2x_n}{3} + \frac{4}{x_n^2},$$

with initial value $x_1 = 2$, converges to α.

(i) Use this iterative formula to determine α correct to 2 decimal places, giving the result of each iteration to 4 decimal places. [3]

(ii) State an equation that is satisfied by α and hence find the exact value of α. [2]

3 **(i)** By sketching a suitable pair of graphs, show that the equation

$$\sec x = 3 - x,$$

where x is in radians, has only one root in the interval $0 < x < \frac{1}{2}\pi$. [2]

(ii) Verify by calculation that this root lies between 1.0 and 1.2. [2]

(iii) Show that this root also satisfies the equation

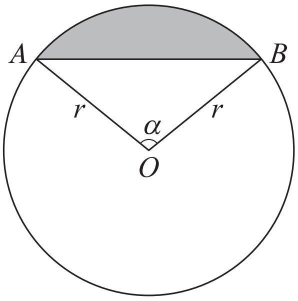
$$x = \cos^{-1}\left(\frac{1}{3 - x}\right).$$ [1]

(iv) Use the iterative formula

$$x_{n+1} = \cos^{-1}\left(\frac{1}{3 - x_n}\right),$$

with initial value $x_1 = 1.1$, to calculate the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

4



The diagram shows a chord joining two points, A and B, on the circumference of a circle with centre O and radius r. The angle AOB is α radians, where $0 < \alpha < \pi$. The area of the shaded segment is one sixth of the area of the circle.

(i) Show that α satisfies the equation

$$x = \tfrac{1}{3}\pi + \sin x.$$ [3]

(ii) Verify by calculation that α lies between $\tfrac{1}{2}\pi$ and $\tfrac{2}{3}\pi$. [2]

(iii) Use the iterative formula

$$x_{n+1} = \tfrac{1}{3}\pi + \sin x_n,$$

with initial value $x_1 = 2$, to determine α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

5 **(i)** By sketching a suitable pair of graphs, show that there is only one value of x that is a root of the equation $x = 9e^{-2x}$. [2]

(ii) Verify, by calculation, that this root lies between 1 and 2. [2]

(iii) Show that, if a sequence of values given by the iterative formula

$$x_{n+1} = \tfrac{1}{2}(\ln 9 - \ln x_n)$$

converges, then it converges to the root of the equation given in part **(i)**. [2]

(iv) Use the iterative formula, with $x_1 = 1$, to calculate the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

6 The sequence of values given by the iterative formula

$$x_{n+1} = \frac{3x_n}{4} + \frac{2}{x_n^3},$$

with initial value $x_1 = 2$, converges to α.

(i) Use this iteration to calculate α correct to 2 decimal places, showing the result of each iteration to 4 decimal places. [3]

(ii) State an equation which is satisfied by α and hence find the exact value of α. [2]