

Functions

Question Paper 4

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Functions
Sub Topic	
Booklet	Question Paper 4

Time Allowed: 50 minutes

Score: /41

Percentage: /100

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	77.5%	70%	62.5%	57.5%	45%	<45%

1 Functions f and g are defined by

$$f : x \mapsto 2x + 5 \quad \text{for } x \in \mathbb{R},$$
$$g : x \mapsto \frac{8}{x-3} \quad \text{for } x \in \mathbb{R}, x \neq 3.$$

- (i) Obtain expressions, in terms of x , for $f^{-1}(x)$ and $g^{-1}(x)$, stating the value of x for which $g^{-1}(x)$ is not defined. [4]
- (ii) Sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the same diagram, making clear the relationship between the two graphs. [3]
- (iii) Given that the equation $fg(x) = 5 - kx$, where k is a constant, has no solutions, find the set of possible values of k . [5]

2 Functions f and g are defined by

$$f : x \mapsto 2x^2 - 8x + 10 \quad \text{for } 0 \leq x \leq 2,$$
$$g : x \mapsto x \quad \text{for } 0 \leq x \leq 10.$$

- (i) Express $f(x)$ in the form $a(x + b)^2 + c$, where a , b and c are constants. [3]
- (ii) State the range of f . [1]
- (iii) State the domain of f^{-1} . [1]
- (iv) Sketch on the same diagram the graphs of $y = f(x)$, $y = g(x)$ and $y = f^{-1}(x)$, making clear the relationship between the graphs. [4]
- (v) Find an expression for $f^{-1}(x)$. [3]

3 The functions f and g are defined for $x \in \mathbb{R}$ by

$$f : x \mapsto 3x + a,$$
$$g : x \mapsto b - 2x,$$

where a and b are constants. Given that $ff(2) = 10$ and $g^{-1}(2) = 3$, find

- (i) the values of a and b , [4]
- (ii) an expression for $fg(x)$. [2]

4 Functions f and g are defined by

$$f : x \mapsto 2x + 3 \quad \text{for } x \leq 0,$$

$$g : x \mapsto x^2 - 6x \quad \text{for } x \leq 3.$$

- (i) Express $f^{-1}(x)$ in terms of x and solve the equation $f(x) = f^{-1}(x)$. [3]
- (ii) On the same diagram sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$, showing the coordinates of their point of intersection and the relationship between the graphs. [3]
- (iii) Find the set of values of x which satisfy $gf(x) \leq 16$. [5]