

# Polynomials

## Factor & Remainder Theorem

### Question Paper 4

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|-------------------|--|
| <b>Level</b>      | International A Level                    |
| <b>Subject</b>    | Maths                                    |
| <b>Exam Board</b> | CIE                                      |
| <b>Topic</b>      | Algebra                                  |
| <b>Sub Topic</b>  | Polynomials – Factor & Remainder Theorem |
| <b>Booklet</b>    | Question Paper 4                         |

**Time Allowed:** 75 minutes

**Score:** /62

**Percentage:** /100

**Grade Boundaries:**

| A*   | A     | B   | C     | D     | E   | U    |
|------|-------|-----|-------|-------|-----|------|
| >85% | 77.5% | 70% | 62.5% | 57.5% | 45% | <45% |

- 1** The polynomial  $2x^3 + 7x^2 + ax + b$ , where  $a$  and  $b$  are constants, is denoted by  $p(x)$ . It is given that  $(x + 1)$  is a factor of  $p(x)$ , and that when  $p(x)$  is divided by  $(x + 2)$  the remainder is 5. Find the values of  $a$  and  $b$ . [5]
- 2** The polynomial  $3x^3 + 8x^2 + ax - 2$ , where  $a$  is a constant, is denoted by  $p(x)$ . It is given that  $(x + 2)$  is a factor of  $p(x)$ .
- (i) Find the value of  $a$ . [2]
- (ii) When  $a$  has this value, solve the equation  $p(x) = 0$ . [4]
- 3** The polynomial  $2x^3 - 3x^2 + ax + b$ , where  $a$  and  $b$  are constants, is denoted by  $p(x)$ . It is given that  $(x - 2)$  is a factor of  $p(x)$ , and that when  $p(x)$  is divided by  $(x + 2)$  the remainder is  $-20$ .
- (i) Find the values of  $a$  and  $b$ . [5]
- (ii) When  $a$  and  $b$  have these values, find the remainder when  $p(x)$  is divided by  $(x^2 - 4)$ . [3]
- 4** The polynomial  $4x^3 - 7x + a$ , where  $a$  is a constant, is denoted by  $p(x)$ . It is given that  $(2x - 3)$  is a factor of  $p(x)$ .
- (i) Show that  $a = -3$ . [2]
- (ii) Hence, or otherwise, solve the equation  $p(x) = 0$ . [4]
- 5** The cubic polynomial  $ax^3 + bx^2 - 3x - 2$ , where  $a$  and  $b$  are constants, is denoted by  $p(x)$ . It is given that  $(x - 1)$  and  $(x + 2)$  are factors of  $p(x)$ .
- (i) Find the values of  $a$  and  $b$ . [5]
- (ii) When  $a$  and  $b$  have these values, find the other linear factor of  $p(x)$ . [2]

- 6 The polynomial  $x^3 - x^2 + ax + b$  is denoted by  $p(x)$ . It is given that  $(x + 1)$  is a factor of  $p(x)$  and that when  $p(x)$  is divided by  $(x - 2)$  the remainder is 12.
- (i) Find the values of  $a$  and  $b$ . [5]
- (ii) When  $a$  and  $b$  have these values, factorise  $p(x)$ . [2]
- 7 The cubic polynomial  $2x^3 - 5x^2 + ax + b$  is denoted by  $f(x)$ . It is given that  $(x - 2)$  is a factor of  $f(x)$ , and that when  $f(x)$  is divided by  $(x + 1)$  the remainder is  $-6$ . Find the values of  $a$  and  $b$ . [5]
- 8 The cubic polynomial  $2x^3 + ax^2 - 13x - 6$  is denoted by  $f(x)$ . It is given that  $(x - 3)$  is a factor of  $f(x)$ .
- (i) Find the value of  $a$ . [2]
- (ii) When  $a$  has this value, solve the equation  $f(x) = 0$ . [4]
- 9 The polynomial  $x^4 - 6x^2 + x + a$  is denoted by  $f(x)$ .
- (i) It is given that  $(x + 1)$  is a factor of  $f(x)$ . Find the value of  $a$ . [2]
- (ii) When  $a$  has this value, verify that  $(x - 2)$  is also a factor of  $f(x)$  and hence factorise  $f(x)$  completely. [4]
- 10 The polynomial  $x^4 - 9x^2 - 6x - 1$  is denoted by  $f(x)$ .
- (i) Find the value of the constant  $a$  for which
- $$f(x) \equiv (x^2 + ax + 1)(x^2 - ax - 1). \quad [3]$$
- (ii) Hence solve the equation  $f(x) = 0$ , giving your answers in an exact form. [3]