Stationary Points

Question Paper 6

Level	International A Level
Subject	Maths
Exam Board	CIE
Topic	Differentiation
Sub Topic	Stationary Points
Booklet	Question Paper 6

Time Allowed: **58 minutes**

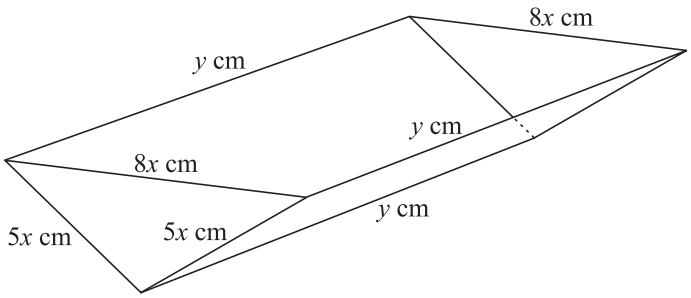
Score: **/48**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	'77.5%	70%	62.5%	57.5%	45%	<45%

1



The diagram shows an open container constructed out of 200 cm^2 of cardboard. The two vertical end pieces are isosceles triangles with sides $5x$ cm, $5x$ cm and $8x$ cm, and the two side pieces are rectangles of length y cm and width $5x$ cm, as shown. The open top is a horizontal rectangle.

(i) Show that $y = \dfrac{200 - 24x^2}{10x}$. [3]

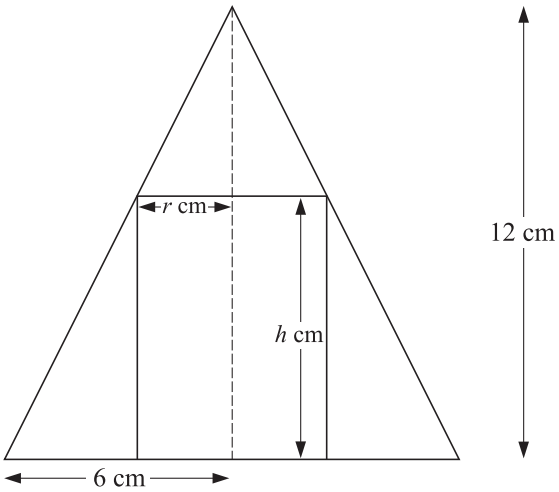
(ii) Show that the volume, V cm^3, of the container is given by $V = 240x - 28.8x^3$. [2]

Given that x can vary,

(iii) find the value of x for which V has a stationary value, [3]

(iv) determine whether it is a maximum or a minimum stationary value. [2]

2



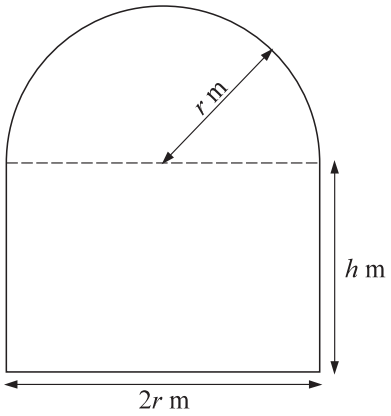
The diagram shows the cross-section of a hollow cone and a circular cylinder. The cone has radius 6 cm and height 12 cm, and the cylinder has radius r cm and height h cm. The cylinder just fits inside the cone with all of its upper edge touching the surface of the cone.

(i) Express h in terms of r and hence show that the volume, V cm^3, of the cylinder is given by

$$V = 12\pi r^2 - 2\pi r^3.$$ [3]

(ii) Given that r varies, find the stationary value of V. [4]

3



The diagram shows a glass window consisting of a rectangle of height h m and width $2r$ m and a semicircle of radius r m. The perimeter of the window is 8 m.

(i) Express h in terms of r. [2]

(ii) Show that the area of the window, A m^2, is given by

$$A = 8r - 2r^2 - \tfrac{1}{2}\pi r^2.$$ [2]

Given that r can vary,

(iii) find the value of r for which A has a stationary value, [4]

(iv) determine whether this stationary value is a maximum or a minimum. [2]

4 A solid rectangular block has a base which measures $2x$ cm by x cm. The height of the block is y cm and the volume of the block is 72 cm^3.

(i) Express y in terms of x and show that the total surface area, A cm^2, of the block is given by

$$A = 4x^2 + \frac{216}{x}.$$ [3]

Given that x can vary,

(ii) find the value of x for which A has a stationary value, [3]

(iii) find this stationary value and determine whether it is a maximum or a minimum. [3]

5 **(i)** Sketch the graph of the curve $y = 3\sin x$, for $-\pi \leqslant x \leqslant \pi$. [2]

The straight line $y = kx$, where k is a constant, passes through the maximum point of this curve for $-\pi \leqslant x \leqslant \pi$.

(ii) Find the value of k in terms of π. [2]

(iii) State the coordinates of the other point, apart from the origin, where the line and the curve intersect. [1]

6 A curve has equation $y = x^3 + 3x^2 - 9x + k$, where k is a constant.

(i) Write down an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$. [2]

(ii) Find the x-coordinates of the two stationary points on the curve. [2]

(iii) Hence find the two values of k for which the curve has a stationary point on the x-axis. [3]